MATH 416: Abstract Linear Algebra

Fall 2025 — University of Illinois, Urbana-Champaign

Instructor: Jacob Beckey

Course Information

Welcome to MATH 416: Abstract Linear Algebra! Linear algebra is an essential component of modern mathematics, physics, and computer science. This course will provide a rigorous treatment of the subject, focusing on the structure and beauty of vector spaces, linear maps, eigenvectors, and much more. In addition to being elegant and powerful, the course content will also provide ample opportunity for us to understand what mathematics is, what mathematicians do, and how they actually do it!

- **Instructor:** Jacob Beckey
- **Grader:** Hyun Tark
- **Meeting times:** Mon, Wed, Fri — 1:00–1:50 PM, 136 Davenport Hall
- **Office Hours:** Mon 2-2:50 PM (214 Davenport Hall), Wed 2-2:50 PM (132 Davenport Hall), or by appointment

Contact & Communication

- **Instructor's email:** jbeckey@illinois.edu
- **Instructor's office:** 72 Computing Applications Building (hopefully Harker Hall soon!)
- **Grader's email:** htark2@illinois.edu

Please reach out if you have questions about course content, assignments, or logistics.

Schedule & Grading

My goal in this course is to help you deeply learn linear algebra while also developing your ability to think abstractly, write cogent proofs, and work collaboratively. Education research has made clear that active engagement increases student learning outcomes and, even more importantly, mathematics is a fundamentally social endeavor. As such, class participation is a significant part of your grade. This grade will be determined based on participation, not correctness, of in-class clicker questions and pre-lecture quizzes. To accommodate unforeseen circumstances, I will drop the lowest two homework grades, the lowest midterm grade, and 5 participation items. With those caveats, your grade will be calculated based on the following breakdown.

Grade Breakdown

- **Participation:** 20%
- **Homework:** 30%
- **Midterm Exams:** 30%
- **Final Exam:** 20%

Homework

Problem sets will be assigned on Fridays and will be due (via Canvas) on the following Friday. LaTeX is ubiquitous in STEM research fields, so I implore all students to use LaTeX for writing up solutions, but this is not a requirement. If you have not ever used LaTeX, I recommend starting with this tutorial.

Exam Dates

- **Midterm Exam 1:** Friday, September 26
- **Midterm Exam 2:** Friday, October 24
- **Midterm Exam 3:** Wednesday, November 19
- **Final Exam:** Wednesday, December 17 (7–10pm, location TBD)

Course Materials

Required Materials

- **Textbook:** Sheldon Axler, *Linear Algebra Done Right* (free PDF available)
- **Technology:** iClicker app (student instructions here)

Strongly Suggested Resources

- Robyn Arianhod, *Vector: A Surprising Story of Space, Time, and Mathematical Transformation*
- Grant Sanderson (3Blue1Brown), Essence of Linear Algebra video series

Course Outline

We will aim to cover most of the content in Chapters 1, 2, 3, 5, 6, 7, and 8 of Axler. This is a lot of material; however, if you work hard in this course, it is hard to overstate how useful it will be in your future. The core topic list will be as follows:

- Solving systems of linear equations
- Complex numbers, vector spaces, subspaces
- Span and linear independence, bases, dimension of vector spaces
- Linear maps, null space and range, matrices, isomorphic vector spaces, isomorphism theorem, dual spaces
- Invariant subspaces, eigenvalues, eigenvectors, upper-triangular matrices
- Inner products, norms, orthonormal bases, orthogonal complements
- Operators on inner product spaces, self-adjoint and normal operators, spectral theorem, isometries, singular value decomposition
- Generalized eigenspaces, multiplicity of eigenvalues, characteristic and minimal polynomial, Jordan normal form
- Trace and determinant

Collaboration and Use of Generative AI

Generative AI tools (such as ChatGPT, Claude, Gemini, Copilot, etc.) and other online resources can be useful for learning, but they must be used responsibly. In this course, the goal of assignments is to help **you** build understanding and practice thinking independently. To that end:

Permitted Uses

- You may use generative AI tools, textbooks, online forums, and other resources to **help you study, brainstorm, or clarify concepts**.
- You may work on homework with classmates, but your submitted work must be your own.

Requirements for Transparency

When you submit homework, you must clearly state all resources you used, including:

- Generative AI tools (e.g., "I wrote this generative AI policy using ChatGPT.").
- Online resources (e.g., "I got started with the proof, but ended up using StackExchange for the crucial step in line 4.").
- Collaborators (e.g., "I worked with Alex and Priya on problem 2.").

If you used an AI tool to generate draft text, code, or problem solutions, you must **explain how you used it** (e.g., "I generated code using GitHub Copilot, but I checked the logic and rewrote sections in my own words.").

What is Not Allowed

- Copying AI-generated or online solutions and submitting them as your own is considered **plagiarism/academic dishonesty**.
- Relying on AI without understanding the reasoning behind a solution defeats the purpose of the assignment and will harm your learning.

Bottom line: You may use AI tools and online resources, but you must **acknowledge them** and ensure that your submitted work reflects your own understanding. Being transparent about your process helps me support your learning and models how academic research is done.

Classroom Climate, Inclusivity, and Anti-Harassment

People learn mathematics best when they feel safe, respected, and comfortable asking questions—including the so-called "dumb" ones (which are often confusions shared by many!). In this course, we are committed to creating a supportive environment where everyone can engage fully in discourse and grow as mathematical thinkers.

Expectations for Our Classroom Community

- All students are expected to treat one another with respect, patience, and kindness in class discussions, group work, and online communication.
- Questions, confusion, and mistakes are a normal and valuable part of learning mathematics. We will celebrate curiosity and persistence, not just quick answers.
- Disruptive, dismissive, or disrespectful behavior (including but not limited to harassment based on race, gender, sexual orientation, religion, ability, or background) will not be tolerated.

Commitment to Inclusivity

- Our classroom welcomes students from all backgrounds and levels of preparation.
- Diversity of perspectives strengthens mathematical conversations, and your contributions are valued.
- If something in the course environment makes it harder for you to participate or feel included, I encourage you to let me know so we can work together to address it.